Exercise 25

For the following exercises, use the Intermediate Value Theorem to confirm that the given polynomial has at least one zero within the given interval.

$$f(x) = x^3 - 9x, \text{ between } x = 2 \text{ and } x = 4.$$

Solution

Plug $x = 2$ and $x = 4$ into the function.

$$f(2) = (2)^3 - 9(2) = -10$$

$$f(4) = (4)^3 - 9(4) = 28$$

Since $f(x)$ is a polynomial function (a smooth and continuous function), $f(x)$ has to take on every value between -10 and 28 for $2 < x < 4$ by the Intermediate Value Theorem. Therefore, $f(x)$ has a zero between $x = 2$ and $x = 4$.